Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Complex Analysis

Subject Code: 4SC05CAC1 **Branch: B.Sc. (Mathematics)**

Semester: 5 Date : 02/12/2015 Time :2:30 To 5:30 Marks :70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1

(A) The function $\bar{z}$ is not analytic at any point. Statement is True or False? (01)

(B) State fundamental theorem of algebra. (01)

(C) Convert in to polar form $f(z) = z + \frac{1}{z}$ and find its real and imaginary components. (02)

(D) Prove that $f(z) = z + \bar{z}$ is real valued function. (02)

(E) State sufficient condition for a function $f(z)$ to be analytic. (02)

(F) Show that $\phi(x, y) = e^x \cos y$ is harmonic function. (02)

(G) Find invariant points for $f(z) = \frac{3z-5}{z+1}$. (02)

(H) Find arc length for the curve $c: z(t) = 1 - 3it, t \in [-1,1]$. (02)

Q-2 **Attempt all questions** **(14)**

(A) Suppose $f(z) = u + iv, z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$, then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x, y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x, y) = v_0$ (06)

(B) If $f(z) = \begin{cases} \frac{ax^3-by^3}{ax^2+by^2} + i\,\frac{ax^3+by^3}{ax^2+by^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$ then prove that C-R equations are satisfied at origin but $f'(z)$ does not exists. (05)

(C) Check whether $\lim_{z\to 0}\frac{\bar{z}}{z}$ exists or not? If it exists, find limit. (03)

Q-3 Attempt all questions (14)

(A) If $f(z) = u(x,y) + i\,v(x,y)$ is differentiable at $z \in \mathbb{C}$ then prove that first order partial derivative of u and v are exist and satisfy the relation $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$. (06)

(B) Prove that $u = x^2 - y^2$ and $v = \frac{-y}{x^2+y^2}$ satisfy Laplace equation. However $f = u + i\,v$ is not analytic function. (04)

(C) Prove that $f(z) = |z|^2$ is not differentiable at $z \neq 0$. (04)

Q-4 Attempt all questions (14)

(A) Let $f(z) = u + iv$ be analytic in domain D then prove that real component u and imaginary component v are harmonic functions. (05)

(B) In usual notation state and prove polar form of C – R equation. (05)

(C) Evaluate: $\int_c \frac{e^z}{(z-3)(z-1)}\,dz$, where c is circle $|z| = 4$. (04)

Q-5 Attempt all questions (14)

(A) Show that $u(x,y) = y^3 - 3x^2y$ is harmonic. Find harmonic conjugate of $u(x,y)$. Also find analytic function. (06)

(B) Find an analytic function $f(z) = u + iv$ such that $u - v = x + y$. (04)

(C) State and prove Liouville’s theorem. (04)

Q-6 Attempt all questions (14)

(A) Find $\int_c z^2\,dz$ where c is a contour which is part of $y = x^2$ from point $z = 0$ to $z = 2 + i$. (05)

(B) State and prove ML – inequality. (05)

(C) Evaluate: $\int_c \frac{z^2 e^z}{(z-1)^3}\,dz$, where c: $|z| = 2$. (04)

Q-7 Attempt all questions (14)

(A) Let c be a simple closed contour in $\mathbb{C}$. Suppose f is analytic within and on c then prove that $\int_c f(z) = 0$. Hence evaluate $\int_c \frac{\sin z}{(z-4)(z-3)}\,dz$, where $c = |z| = 1$. (07)

(B) If four points z_1, z_2, z_3, z_4 of the $z-$plane map on to the points w_1, w_2, w_3, w_4 of the $W-$plane respectively under the bilinear transformation then prove that (07)

$\frac{(w_1-w_2)(w_3-w_4)}{(w_1-w_4)(w_3-w_2)} = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_2)}$. Also Find bilinear transformation that maps the point $z_1 = -1, z_2 = 0, z_3 = 1$ on to $w_1 = -1, w_2 = -i, w_3 = 1$respectively.

Q-8 **Attempt all questions** (14)

(A) Prove that $|z| = r$ in Z – plane transform to a circle in $W -$ plane using transformation $w = z + (3 + 4i)$. Also draw its roughsketch. (05)

(B) Find image of $|z + 1| = 1$ under the transformation $\frac{1}{z}$ and draw its rough sketch. (05)

(C) Prove that $\left|\int_c \frac{1}{z^2-1}\, dz\right| \leq \frac{2\pi}{3}$, where c: $|z| = 2$ is upper half of the circle. (04)